Delayed Stability and Performance of Distributed Congestion Control

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Outline

- Introduction
  - Stability and Delays
  - Classic Kelly Control
- Max-min Kelly Control (MKC)
- Performance
- Implementation
- Conclusion

background (Introduction, Stability and Delays, Classic Kelly Control)

our work (Max-min Kelly Control, Performance, Implementation, Conclusion)

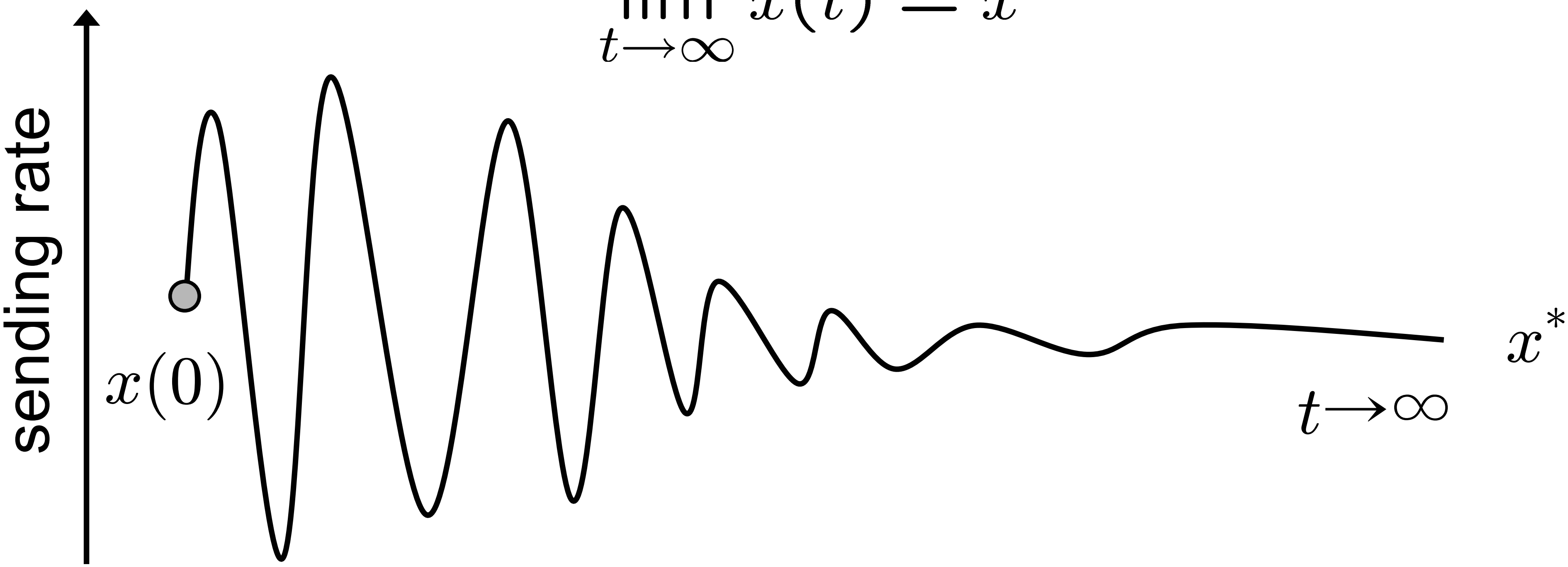
Stability and Delays

- Future high-speed networks are likely to require new types of congestion control
  - Current efforts include XCP, BIC-TCP, FAST TCP, HSTCP, Scalable TCP, etc.
- Besides improving classical E2E approaches, another direction is to involve Active Queue Management (AQM)
  - In AQM, routers compute explicit feedback
  - No per flow management is usually allowed
  - Feedback is computed based on <u>aggregate</u> arrival rates of all flows

Stability and Delays 2

- In AQM congestion control, asymptotic stability is one of the most basic requirements
  - Assume $x(t)$ is the sending rate of a flow at time t
  - Desired behavior:

$$\lim_{t \to \infty} x(t) = x^*$$



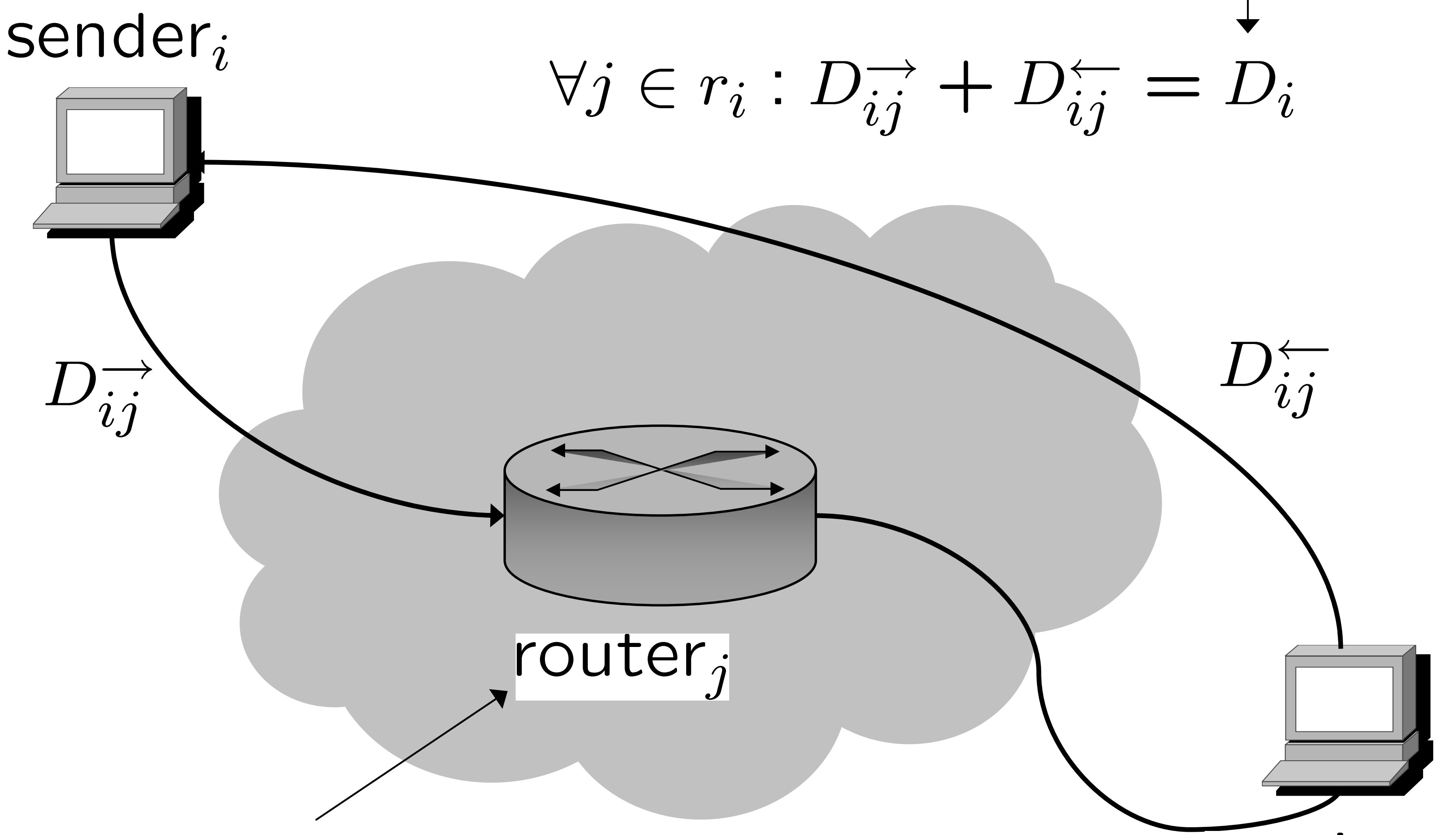
Stability and Delays 3

- Stability is often compromised by feedback delay
- Delayed stability proofs are generally complicated, especially under heterogeneous delay:
  - Each flow has a different RTT equal to D_i time units
  - Metric D_i can be fixed for each flow or changing over time (i.e., random)
- Not only are real Internet delays heterogeneous, they are also directional
  - Delays to/from each router are non-negligible

Stability and Delays 4

RTT

$$\forall j \in r_i : D_{ij}^{\rightarrow} + D_{ij}^{\leftarrow} = D_i$$



For each router j in user's path r_i

Stability and Delays 5

- Does stability under homogeneous delay imply that under heterogeneous delay?
  - The answer is NO
- Until 2004, no prior work obtained a controller that was provably stable under <u>heterogeneous</u> delay and whose stability condition did <u>not</u> depend on delay
  - This paper and Srikant's paper in ACC 2004 are the first two approaches to do so

Classic Kelly Control

- Our analysis examines optimization-based framework introduced by Kelly *et al*. in 1998
  - Kelly control offers an economic interpretation of the user-resource model
  - Performance of the system is optimized when the utilities of end-users are locally maximized
- Continuous control has been proven to be globally asymptotically stable in the absence of delay (Kelly 1998)
  - Further analysis under delay has become an active research field (Massoulie 2002, Kunniyur 2000, 2001, 2003, Vinnicombe 2000, 2002, etc.)

Classic Kelly Control 2

- Stability of Kelly control in the discrete case is studied by Johari in 2001
  - Since all real networks are discrete, we also take this approach

- Under heterogeneous feedback delays, Johari *et al.* discretize Kelly control as follows:

$$x_i(n) = x_i(n-1) + \kappa_i\Big(\omega_i - x_i(n-D_i) \sum_{j \in r_i} \mu_j(n - D_{\overleftarrow{ij}})\Big),$$

- $x_i(n)$: next sending rate
- $x_i(n-1)$: preceding sending rate
- κ_i, ω_i: positive constants
- $x_i(n-D_i)$: rate RTT time units earlier
- $\sum_{j \in r_i}$: over all routers in the path
- μ_j: packet loss of router j

Classic Kelly Control 3

- Under constant delay $D_i = D$, the discrete Kelly control is asymptotically stable if (Johari 2001):

$$\kappa_i \sum_{j \in r_i} ((p_j + p_j' \sum_{u \in s_j} x_u)|_{x_u^*}) < 2 \sin\left(\frac{\pi}{2(2D-1)}\right),$$

where x_u^* is the steady-state rate of user u

- Under heterogeneous delays, the continuous Kelly control is stable if (Vinnicombe 2000):

$$\kappa_i \sum_{j \in r_i} ((p_j + p_j' \sum_{u \in s_j} x_u)|_{x_u^*}) < \frac{\pi}{2D_i}$$

cannot support arbitrarily large delay!

Our Contributions

- Prove that there exists a wide class of non-linear max-min control systems, whose heterogeneous stability does not depend on delays
- Propose a new controller called Max-min Kelly Control (MKC), which is stable under heterogeneous delays, exponentially convergent to efficiency, and quickly convergent to fairness
- Provide novel implementation of AQM congestion control that properly estimates aggregate user rates and achieves theoretically predicted performance

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background

our work

Max-min Kelly Control (MKC)

- End-user equation:

$$x_i(n) = (1 - \beta\eta_i(n))x_i(n - D_i) + \alpha$$

constant (β) — packet loss ($\eta_i(n)$) — rate RTT time units earlier ($x_i(n - D_i)$) — constant (α)

- Utilize max-min fairness, where the feedback is the packet loss of the most-congested resource along the path:

$$\eta_i(n) = \max_{j \in r_i} p_j(n - D_{ij}^{\leftarrow}),$$

set of routers in the path (r_i)

where:

$$p_j(n) = p_j\Big(\sum_{u \in s_j} x_u(n - D_{uj}^{\rightarrow})\Big)$$

aggregate rate ($\sum_{u \in s_j} x_u(n - D_{uj}^{\rightarrow})$)

Delay-Independent Stability

- Theorem. Assume an N-dimensional undelayed nonlinear system $\mathcal{N}$:

$$x_i(n) = f_i(x_1(n-1), x_2(n-1), \cdots, x_N(n-1)),$$

  where $f_i(.)$ are some non-linear functions.

- If the Jacobian matrix J is real-valued and symmetric, then system $\mathcal{N}_{\mathcal{D}}$ with arbitrary delays:

$$x_i(n) = f_i\Big(x_1(n - D_1^{\rightarrow} - D_i^{\leftarrow}), x_2(n - D_2^{\rightarrow} - D_i^{\leftarrow}), \cdots, x_N(n - D_N^{\rightarrow} - D_i^{\leftarrow})\Big)$$

  is stable if and only if $\mathcal{N}$ is stable

Stability of MKC

- The Jacobian of MKC real and symmetric
- <u>Theorem</u>. Heterogeneously delayed MKC is locally asymptotically stable if and only if:

$$0 < \beta p^* < 2,$$

$$0 < \beta p^* + \beta N x^* \left.\frac{\partial p}{\partial x_i}\right|_{\mathbf{x}^*} < 2,$$

stationary packet loss (p^*)

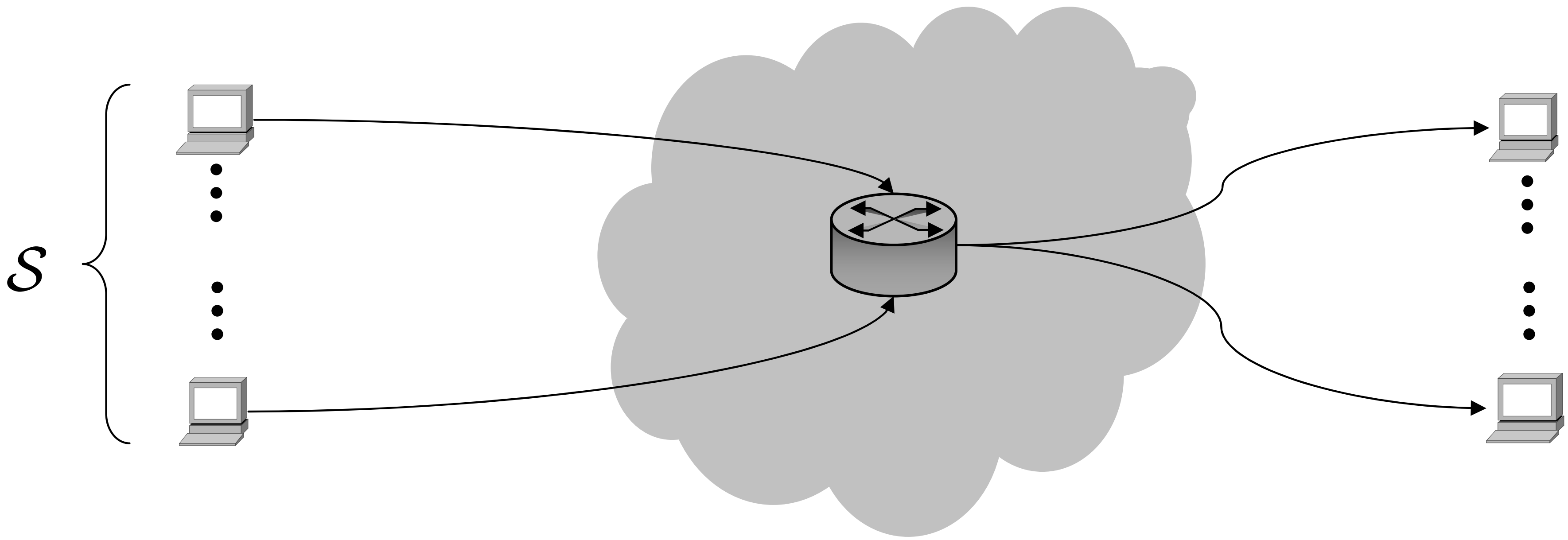
stationary sending rate (x^*)

derivative of the packet loss function ($\left.\frac{\partial p}{\partial x_i}\right|_{\mathbf{x}^*}$)

Stability conditions do not depend on any delays or the routing matrix of end-flows!

Exponential MKC (EMKC)

- Assume a set $\mathcal{S}$ of N users congested by a common link of capacity C



- EMKC has a particular packet loss function $p(n)$:

$$p(n) = \frac{\sum_{u=1}^{N} x_u(n - D_u^{\rightarrow}) - C}{\sum_{u=1}^{N} x_u(n - D_u^{\rightarrow})}$$

Exponential MKC (EMKC) 2

- Theorem. Heterogeneously delayed EMKC is locally asymptotically stable if and only if $0 < \beta < 2$

The only parameter affecting heterogeneous stability of EMKC is β

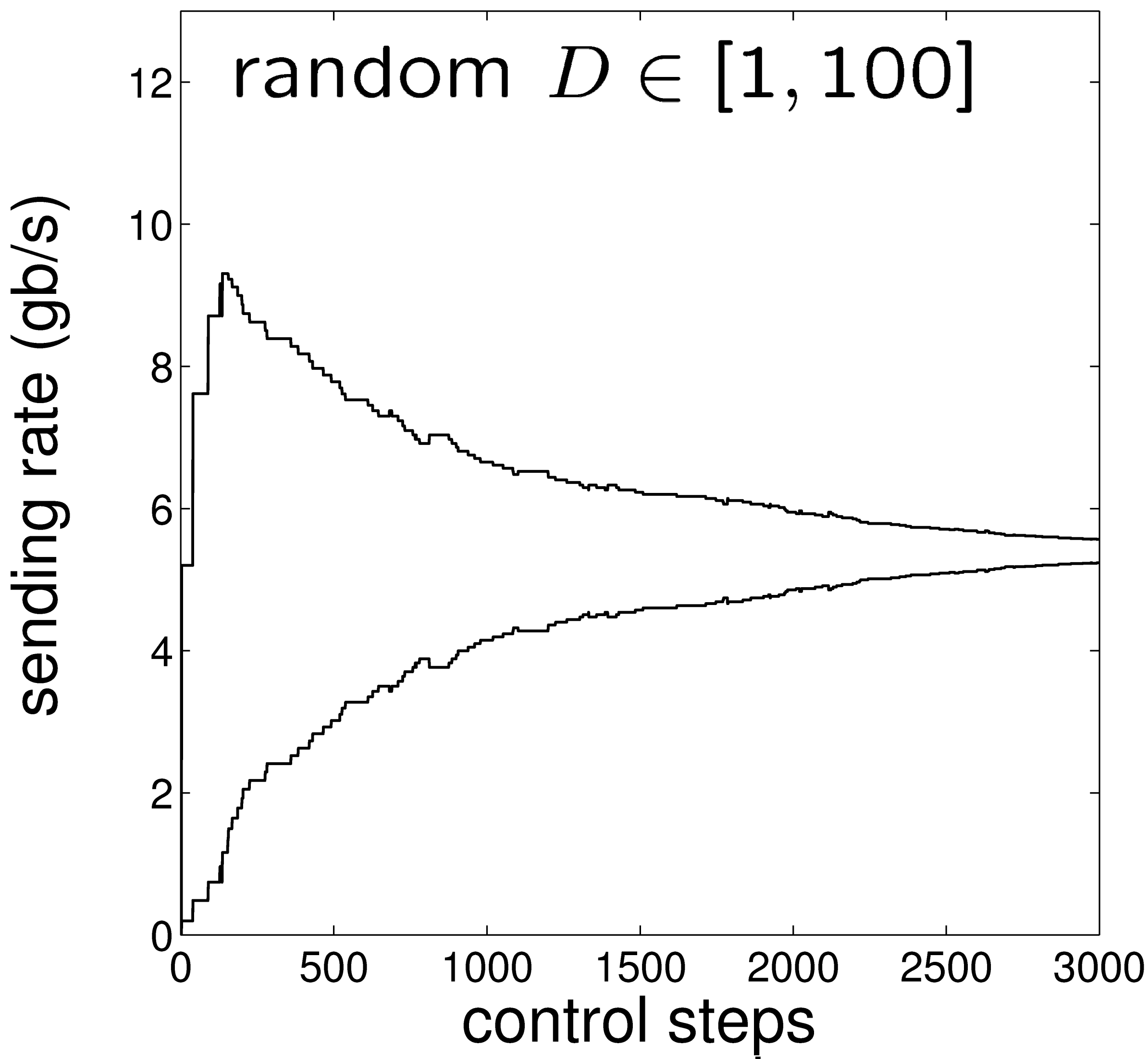
- In fact, many other systems with a symmetric Jacobian exhibit similar delay-independent stability
- The equilibrium individual rate is $x^{*} = C/N + \alpha/\beta$

EMKC is fair regardless of end-flow RTTs!

Exponential MKC (EMKC) 3

- Dynamics of EMKC under constant and random delays

For the same parameters, Kelly control is unstable for $D > 3$



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Convergence to Efficiency

- Under constant delay D, we have:

aggregate flow rate ⟶

$$X(n) = (1-\beta)^{n/D}(X_0 - X^*) + X^*,$$

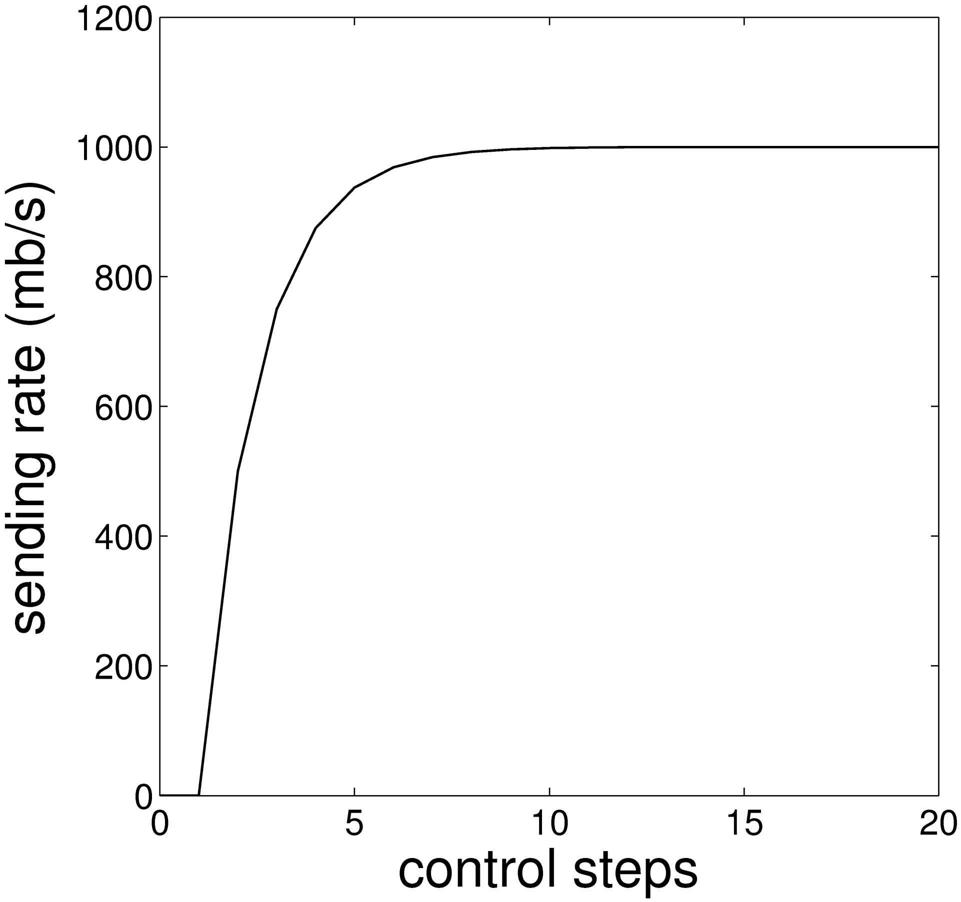
where X_0 is the combined initial rate

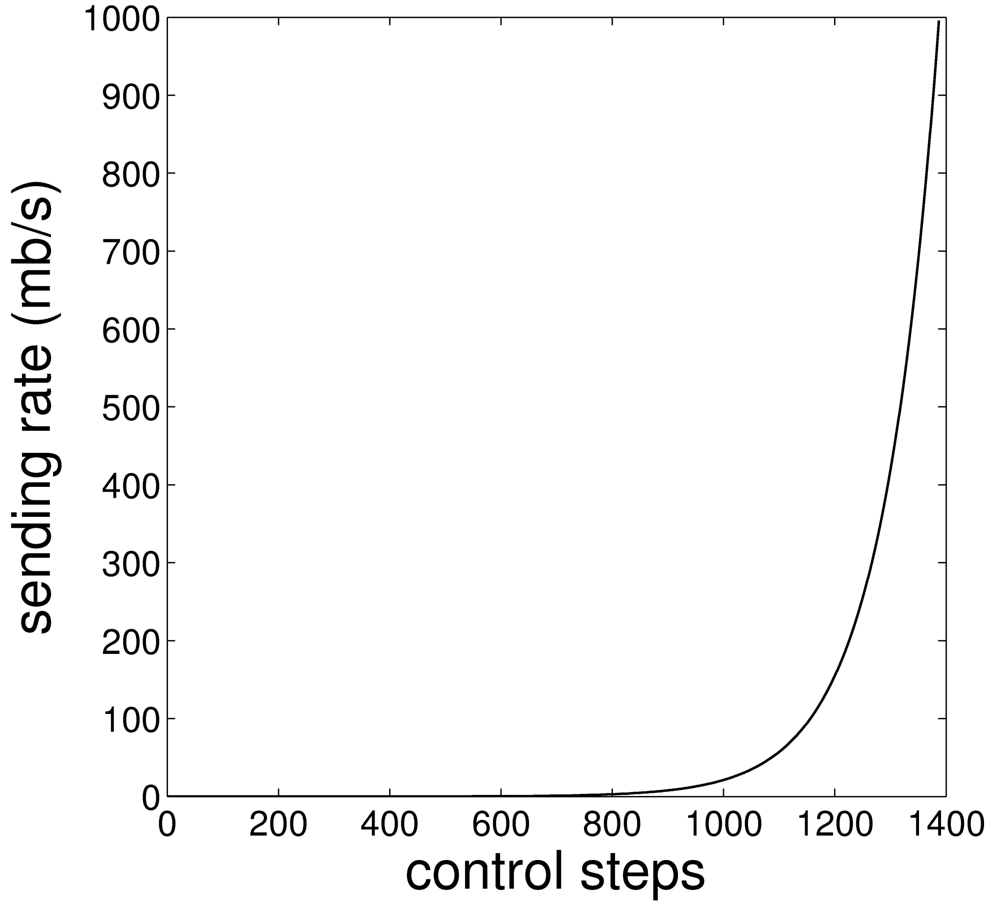
- Lemma. For $0 < \beta < 2$ and constant delay D, the combined rate $X(n)$ of $E_M K_C$ is globally asymptotically stable and converges to X^* at an exponential rate

- For $0 < \beta \leq 1$, $E_M K_C$ monotonically converges to its equilibrium; for $1 < \beta < 2$, $E_M K_C$ experiences decaying oscillations (see paper for examples)

EMKC vs. Scalable TCP

EMKC 1 gb/s link Scalable TCP





EMKC's convergence is 140 times faster than that of Scalable TCP

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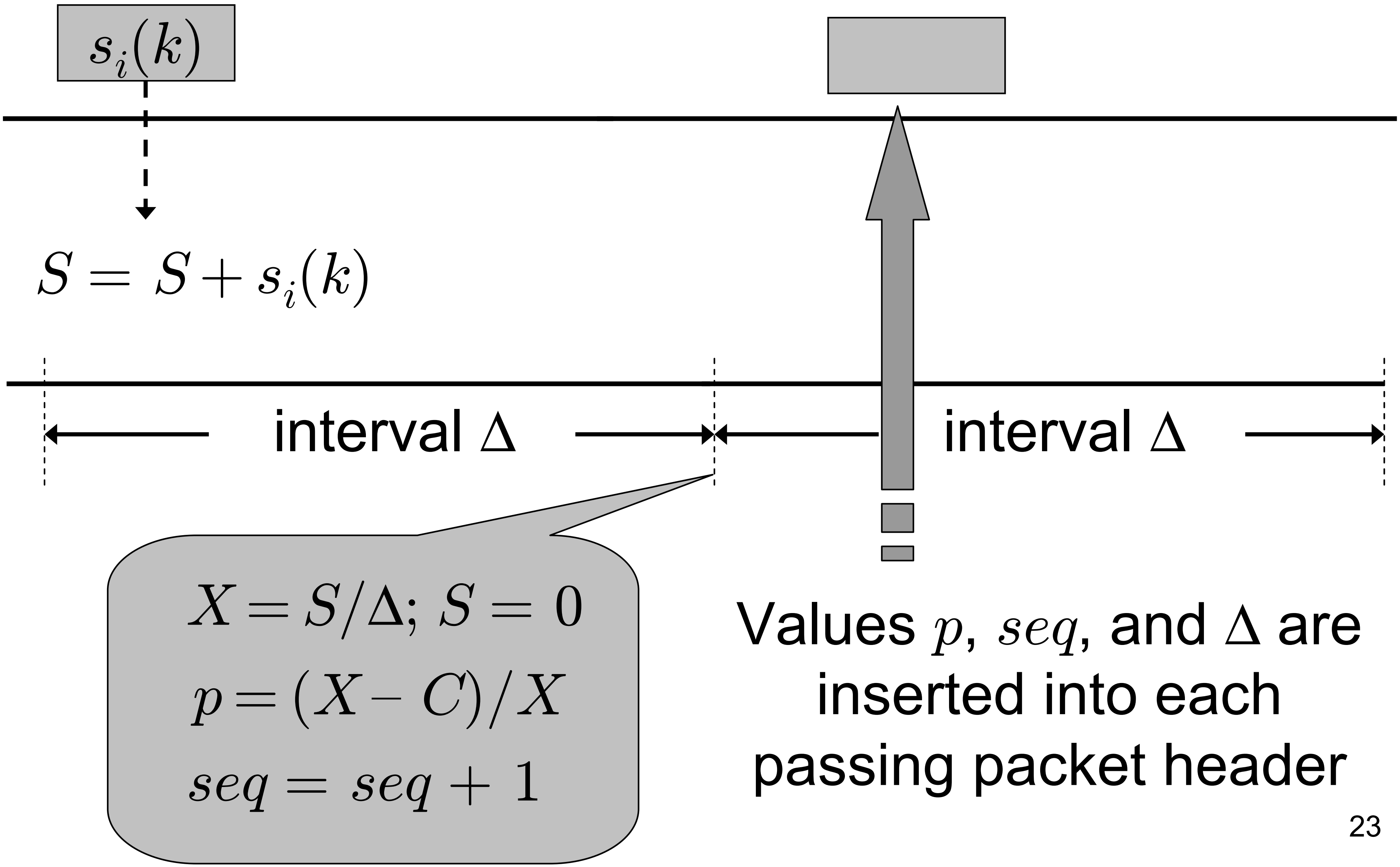
Router

one addition per arriving packet!

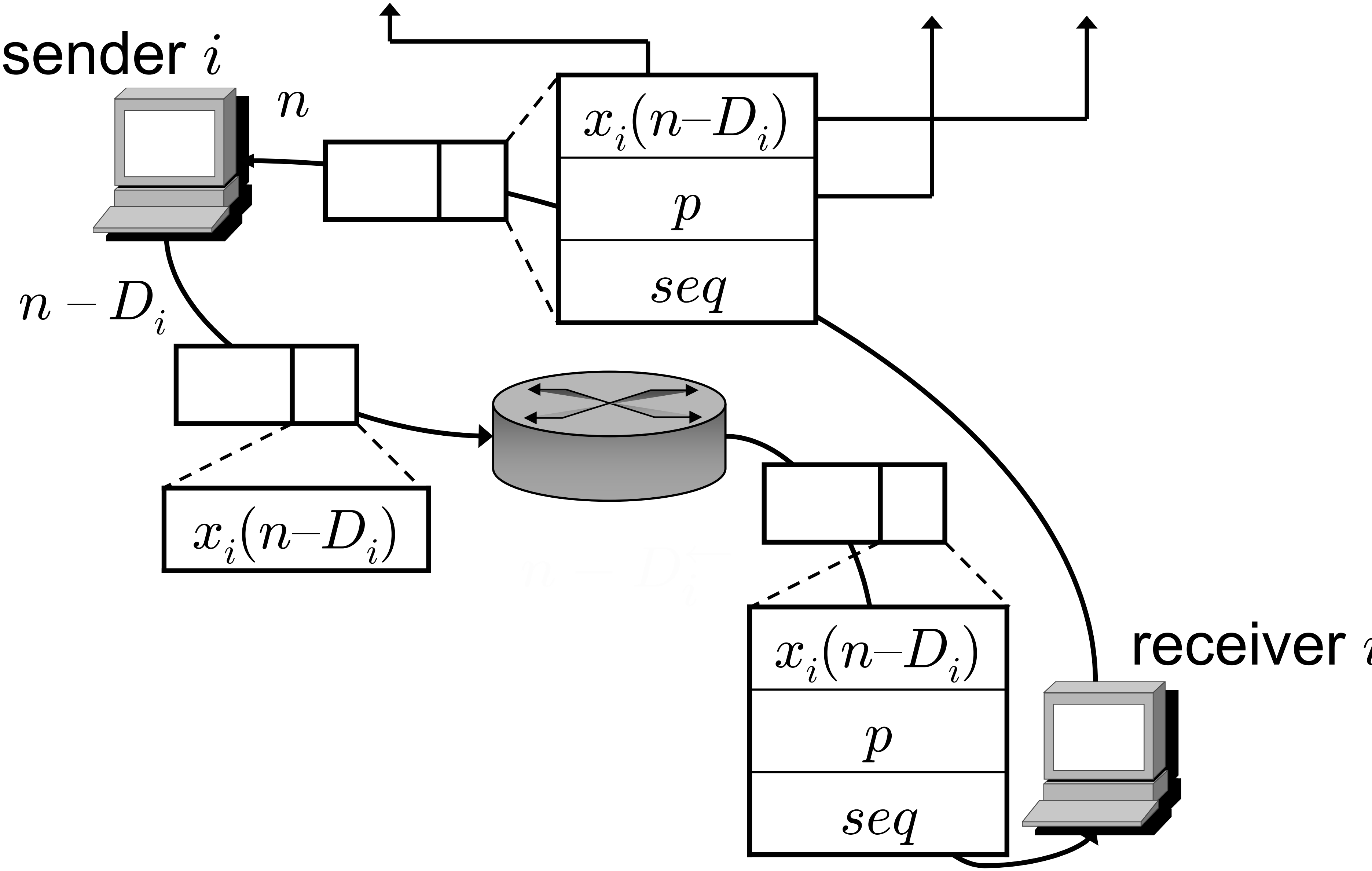


User

- Each ACK carries feedback information
  - To prevent the user from responding to duplicate or reordered packet loss, the sender reacts to each feedback only once (using field seq)
- Recall that MKC requires both the delayed feedback $\eta(n)$ and the delayed reference rate $x_i(n-D_i)$
  - We have the following two options to implement this mechanism

Naïve Implementation

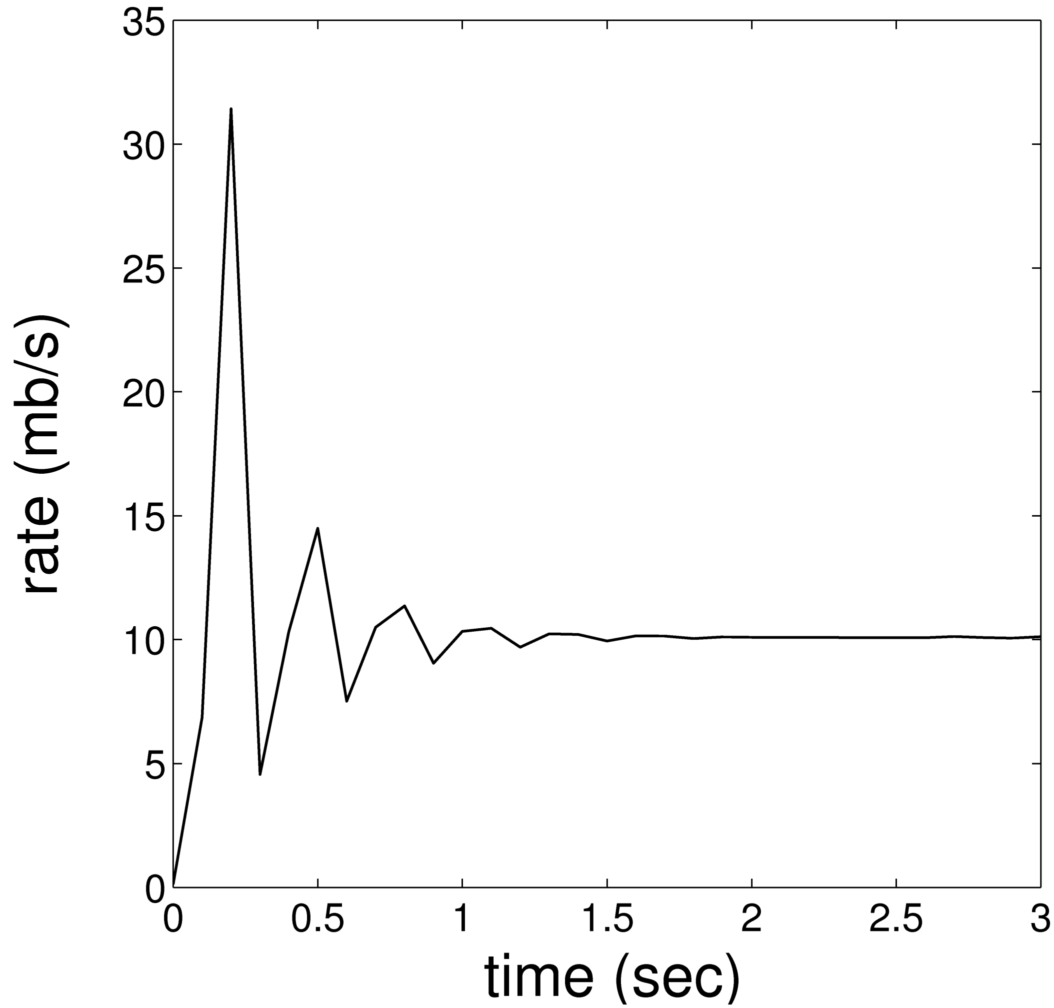
$$x_i(n) = x_i(n - D_i) + \alpha - \beta\eta_i(n)x_i(n - D_i)$$



Naïve Implementation 2

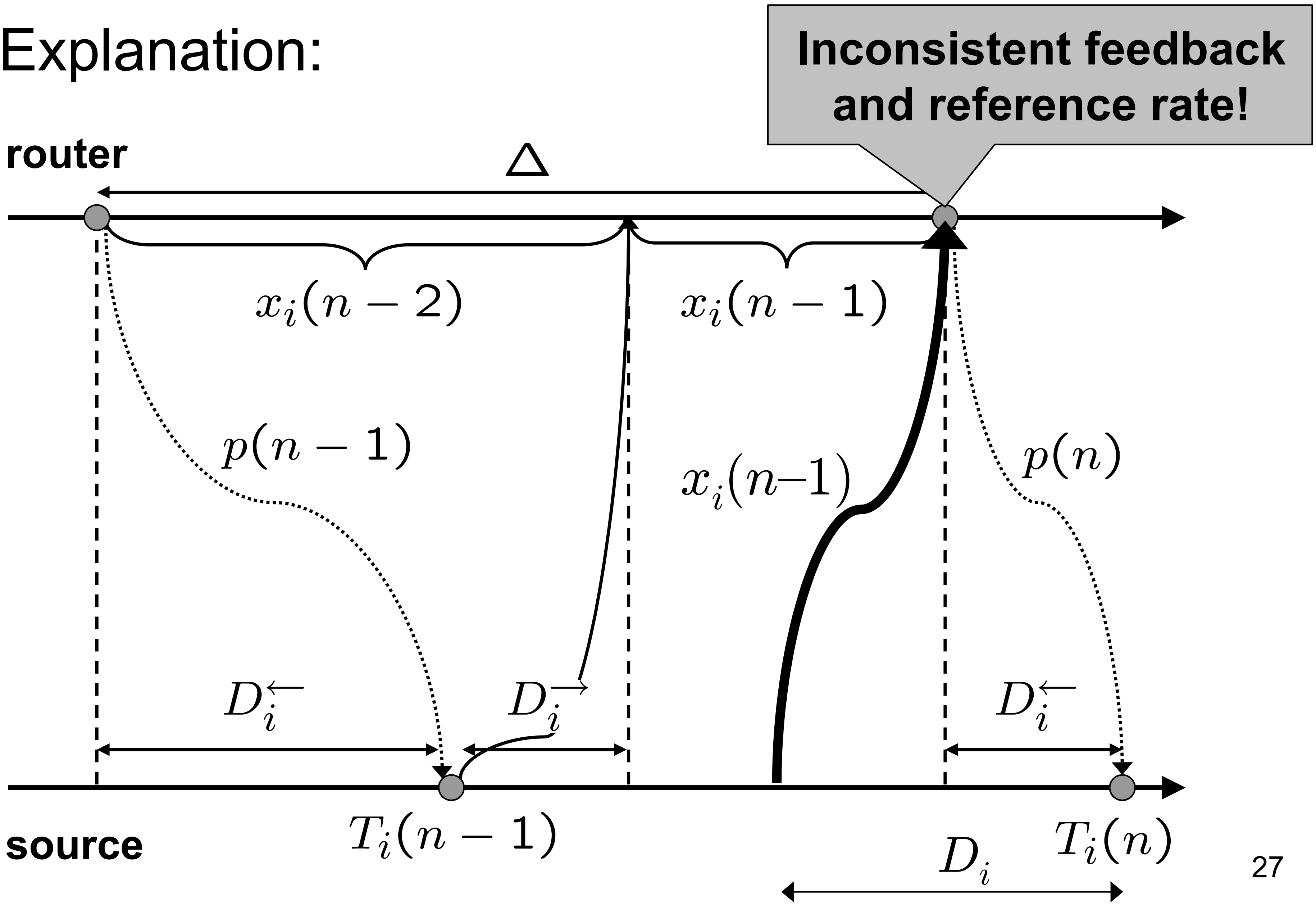
- Consider an `ns2` simulation where $\alpha = 100$ kb/s, $\beta = 0.9$ and $C = 10$ mb/s

Transient overshoot by 200%



Naïve Implementation 3

Explanation:

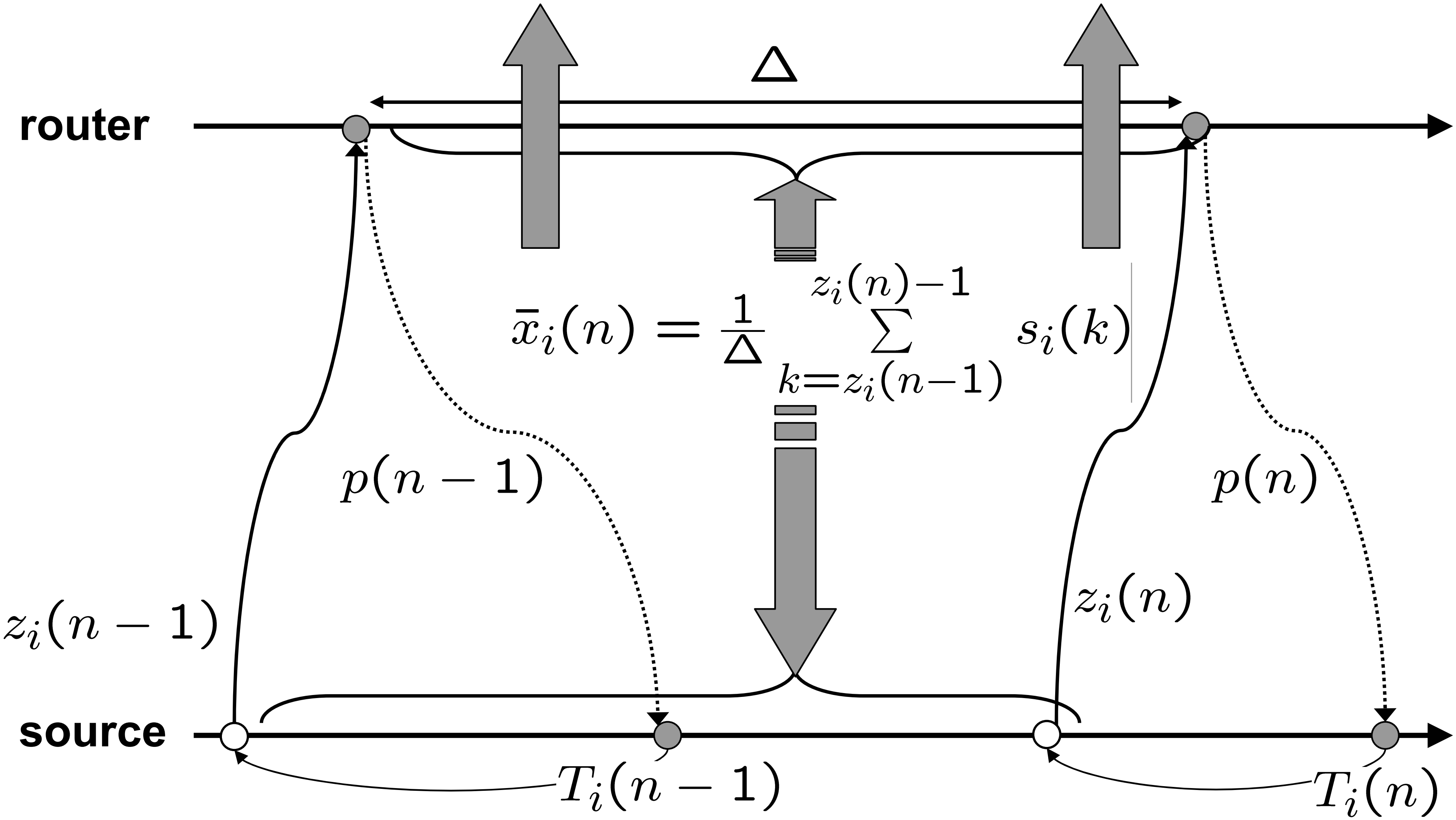


Proper Implementation

- The goal is to make the feedback and the reference rate consistent without extra requirements on the router
- To accomplish this, the sender places a packet sequence number in each out-going packet
- The sender records in the local memory the sequence numbers and sizes of all packets that have been sent since the last rate change

Proper Implementation 2

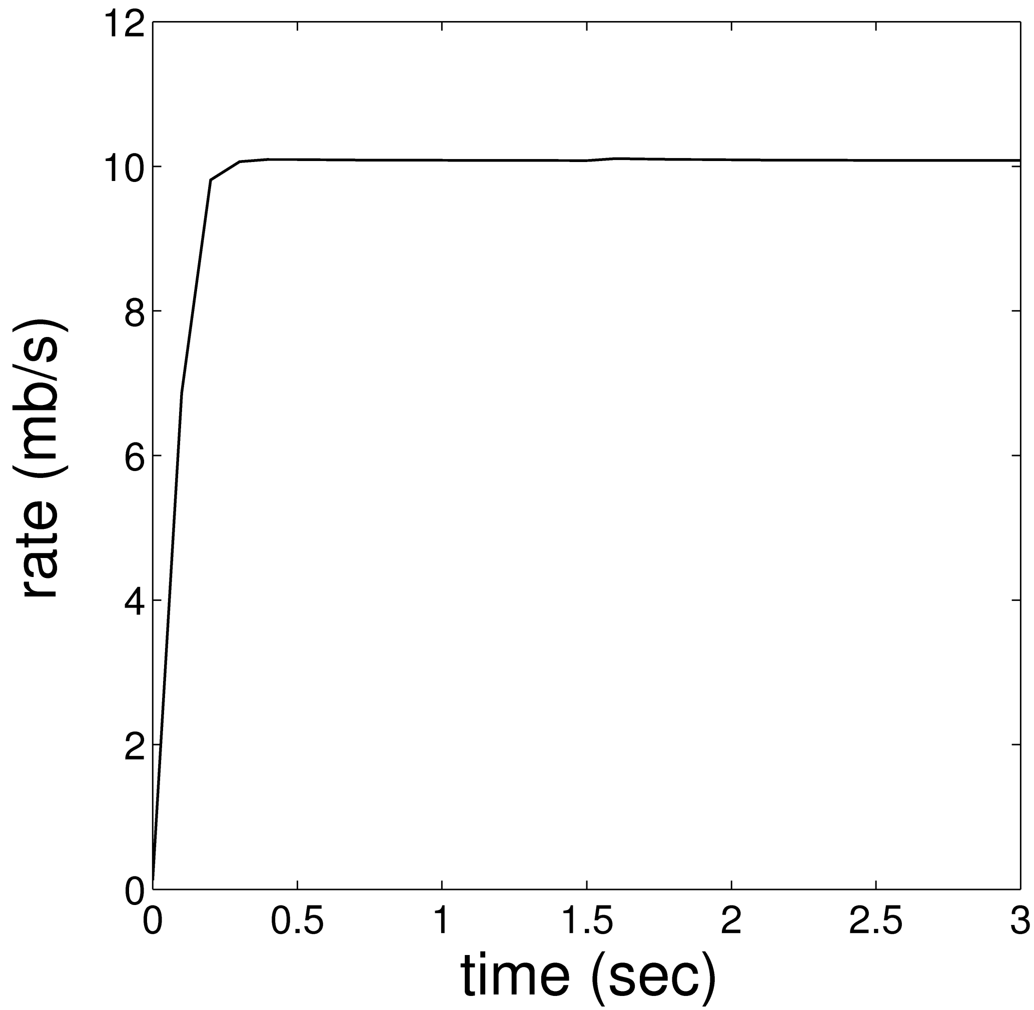
$$x_i(n) = \bar{x}_i(n) + \alpha - \beta\eta_i(n)\bar{x}_i(n)$$



Proper Implementation 3

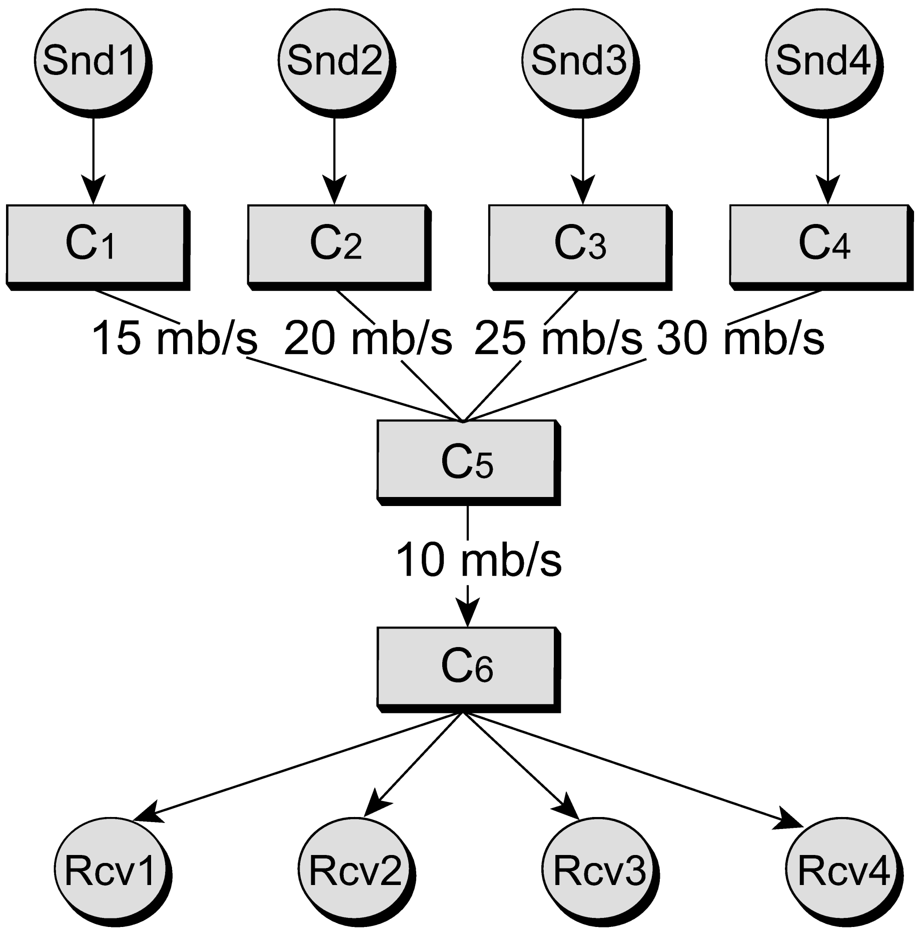
- Consider an ns2 simulation where $\alpha = 100$ kb/s, $\beta = 0.9$, and $C = 10$ mb/s

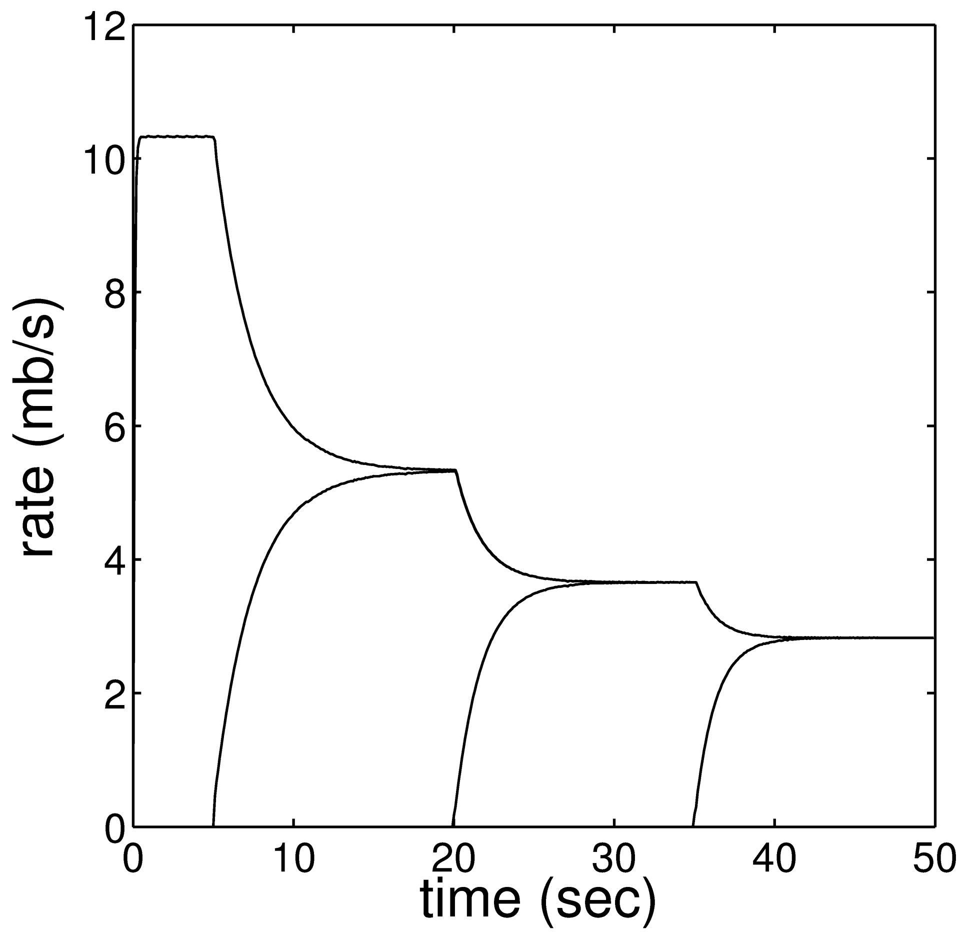
Instantaneous rates (no smoothing)!



Simulation

- Next, we examine $E_M K_C$ under heterogeneous delays in ns2 ($\alpha = 100$ kb/s and $\beta = 0.9$)





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Conclusion

- Heterogeneously stable and discrete AQM congestion control is possible with a very simple implementation and properties desirable in future high-speed networks:
  - Exponential convergence to link utilization
  - Fast convergence to RTT-independent fairness
  - Low overhead